Concept Reinforcement Insert the symbol $<,>, \leq$, or $\geq$ to make each pair of inequalities equivalent.

1. $-5 x \leq 30 ; x \square-6 \geq$
2. $-7 t \geq 56 ; t \square-8 \leq$
3. $-2 t>-14$; $t \square 7$
4. $-3 x<-15$; $x \square 5$
5. $\frac{1}{3} x<-6$; $x \square-18$
6. $9 x \geq-90 ; x \square-10$

Classify each pair of inequalities as "equivalent" or "not equivalent."
7. $x<-2 ;-2>x$
8. $t>-1 ;-1<t$
Equivalent $-4 x-1 \leq 15$;
10. $-2 t+3 \geq 11$;
9. $\begin{aligned} & -4 x-1 \leq 15 ; \\ & -4 x \leq 16 \quad \text { Equivalent }\end{aligned}$
$-2 t \geq 14$ Not equivalent

Determine whether the given numbers are solutions of the inequality.

## 11. $x-4 \geq 1$

a) -4 No
b) 4 No
c) $5 \quad \mathrm{Yes}$
d) $8 \quad \mathrm{Yes}$
12. $3 x+1 \leq-5$
a) -5 Yes
b) -2 Yes
c) 0 No
d) 3 No
13. $2 y+3<6-y$
a) 0 Yes
b) 1 No
c) -1 Yes
d) 4 No
14. $5 t-6>1-2 t$
a) 6 Yes
b) 0 No
c) -3 No
d) 1 No

Graph each inequality, and write the solution set using both set-builder notation and interval notation.
15. $y<6$
16. $x>4$
17. $x \geq-4$
18. $t \leq 6$
19. $t>-3$
20. $y<-3$
21. $x \leq-7$
22. $x \geq-6$

Solve. Then graph. Write the solution set using both setbuilder notation and interval notation.
23. $x+2>1$
24. $x+9>6$
25. $t-6 \leq 4$
26. $t-1 \geq 5$
27. $x-12 \geq-11$
28. $x-11 \leq-2$
29. $9 t<-81$
30. $8 x \geq 24$
$\square$ Answers to Exercises 15-52 and 58 are on pp. IA-8 and IA-9.
31. $-0.3 x>-15$
32. $-0.5 x<-30$
33. $-9 x \geq 8.1$
34. $-8 y \leq 3.2$
35. $\frac{3}{4} y \geq-\frac{5}{8}$
36. $\frac{5}{6} x \leq-\frac{3}{4}$
37. $3 x+1<7$
38. $2 x-5 \geq 9$
39. $3-x \geq 12$
40. $8-x<15$
41. $\frac{2 x+7}{5}<-9$
42. $\frac{5 y+13}{4}>-2$
43. $\frac{3 t-7}{-4} \leq 5$
44. $\frac{2 t-9}{-3} \geq 7$
45. $\frac{9-x}{-2} \geq-6$
46. $\frac{3-x}{-5}<-2$
47. Let $f(x)=7-3 x$ and $g(x)=2 x-3$. Find all values of $x$ for which $f(x) \leq g(x)$.
48. Let $f(x)=8 x-9$ and $g(x)=3 x-11$. Find all values of $x$ for which $f(x) \leq g(x)$.
49. Let $f(x)=2 x-7$ and $g(x)=5 x-9$. Find all values of $x$ for which $f(x)<g(x)$.
50. Let $f(x)=0.4 x+5$ and $g(x)=1.2 x-4$. Find all values of $x$ for which $g(x) \geq f(x)$.
51. Let $y_{1}=\frac{3}{8}+2 x$ and $y_{2}=3 x-\frac{1}{8}$. Find all values of $x$ for which $y_{2} \geq y_{1}$.
52. Let $y_{1}=2 x+1$ and $y_{2}=-\frac{1}{2} x+6$. Find all values of $x$ for which $y_{1}<y_{2}$.

Solve. Write the solution set using both set-builder notation and interval notation.
53. $3-8 y \geq 9-4 y \quad\left\{y \left\lvert\, y \leq-\frac{3}{2}\right.\right\}$, or $\left(-\infty,-\frac{3}{2}\right]$
54. $4 m+7 \geq 9 m-3 \quad\{m \mid m \leq 2\}$, or $(-\infty, 2]$
55. $5(t-3)+4 t<2(7+2 t) \quad\left\{t \left\lvert\, t<\frac{29}{5}\right.\right\}$, or $\left(-\infty, \frac{29}{5}\right)$
56. $2(4+2 x)>2 x+3(2-5 x) \quad\left\{x \left\lvert\, x>-\frac{2}{17}\right.\right\}$, or $\left(-\frac{2}{17}, \infty\right)$
57. $5[3 m-(m+4)]>-2(m-4) \quad\left\{m \left\lvert\, m>\frac{7}{3}\right.\right\}$, or $\left(\frac{7}{3}, \infty\right)$
58. $8 x-3(3 x+2)-5 \geq 3(x+4)-2 x$
59. $19-(2 x+3) \leq 2(x+3)+x \quad\{x \mid x \geq 2\}$, or $[2, \infty)$
60. $13-(2 c+2) \geq 2(c+2)+3 c \quad\{c \mid c \leq 1\}$, or $(\quad 1]$
61. $\frac{1}{4}(8 y+4)-17<-\frac{1}{2}(4 y-8) \quad\{y \mid y<5\}$, or $(-\infty, 5)$
62. $\frac{1}{3}(6 x+24)-20>-\frac{1}{4}(12 x-72) \quad\{x \mid x>6\}$, or $(6, \infty)$
63. $2[8-4(3-x)]-2 \geq 8[2(4 x-3)+7]-50$
64. $5[3(7-t)-4(8+2 t)]-20 \leq-6[2(6+3 t)-4]$

$$
\left\{t \left\lvert\, t \geq-\frac{27}{19}\right.\right\} \text {, or }\left[-\frac{27}{19}, \infty\right)
$$

Translate to an inequality.
65. A number is less than 10.

Let $n$ represent the number; $n<10$
66. A number is greater than or equal to 4 .

Let $n$ represent the number; $n \geq 4$
67. The temperature is at most $-3^{\circ} \mathrm{C}$.

Let $t$ represent the temperature; $t \leq-3$
68. The credit-card debt of the average college freshman is at least $\$ 2000$.
Let $d$ represent the average credit-card debt; $d \geq 2000$
69. The age of the Mayan altar exceeds 1200 years. Let $a$ represent the age of the altar; $a>1200$
70. The time of the test was between 45 min and 55 min . Let $t$ represent the time of the test; $45<t<55$
71. Normandale Community College is no more than 15 mi away. Let $d$ represent the distance to Normandale Community College; $d \leq 15$
72. Angenita earns no less than $\$ 12$ per hour.

Let $h$ represent Angenita's hourly wage; $h \geq 12$
73. To rent a car, a driver must have a minimum of 5 years of driving experience.
Let $d$ represent the number of years of driving experience; $d \geqq 5$
74. The maximum safe-exposure limit of formaldehyde is 2 parts per million.
Let $f$ represent the amount of exposure to formaldehyde; $f \leq 2$
75. The costs of production of the software cannot exceed $\$ 12,500$.
Let $c$ represent the cost of production; $c \leq 12,500$
76. The cost of gasoline was at most $\$ 4$ per gallon. Let $c$ represent the cost of gasoline; $c \leq 4$

Use an inequality and the five-step process to solve each problem.
Solve.
77. Photography. Eli will photograph a wedding for a flat fee of $\$ 900$ or for an hourly rate of $\$ 120$. For what lengths of time would the hourly rate be less expensive? Lengths of time less than $7 \frac{1}{2} \mathrm{hr}$
78. Truck Rentals. Jenn can rent a moving truck for either $\$ 99$ with unlimited mileage or $\$ 49$ plus $80 \phi$ per mile. For what mileages would the unlimited mileage plan save money? Mileages greater than 62.5 mi
79. Graduate School. An unconditional acceptance into the Master of Business Administration (MBA) program at Arkansas State University will be given to students whose GMAT score plus 200 times the undergraduate grade point average is at least 950 . Chole's GMAT score was 500 . What must her grade point average be in order to be unconditionally accepted into the program? At least 2.25
Source: graduateschool.astate.edu
80. Car Payments. As a rule of thumb, debt payments (other than mortgages) should be less than $8 \%$ of a consumer's monthly gross income. Oliver makes
$\$ 54,000$ per year and has a $\$ 100$ student-loan payment every month. What size car payment can he afford? Less than $\$ 260$
Source: money.cnn.com

81. Exam Scores. There are 80 questions on a college entrance examination. Two points are awarded for each correct answer, and one-half point is deducted for each incorrect answer. How many questions does Tami need to answer correctly in order to score at least 100 on the test? Assume that Tami answers every question. At least 56 questions correct
82. Insurance Claims. After a serious automobile accident, most insurance companies will replace the damaged car with a new one if repair costs exceed $80 \%$ of the NADA, or "blue-book," value of the car. Lorenzo's car recently sustained $\$ 9200$ worth of damage but was not replaced. What was the bluebook value of his car? $\$ 11,500$ or more
83. Well Drilling. All Seasons Well Drilling offers two plans. Under the "pay-as-you-go" plan, they charge $\$ 500$ plus $\$ 8$ per foot for a well of any depth. Under their "guaranteed-water" plan, they charge a flat fee of $\$ 4000$ for a well that is guaranteed to provide adequate water for a household. For what depths would it save a customer money to use the pay-as-you-go plan? Depths less than 437.5 ft
84. Legal Fees. Bridgewater Legal Offices charges a $\$ 250$ retainer fee for real estate transactions plus $\$ 180$ per hour. Dockside Legal charges a $\$ 100$ retainer fee plus $\$ 230$ per hour. For what number of hours does Bridgewater charge more? For fewer than 3 hr
85. Wages. Toni can be paid in one of two ways:

Plan A: A salary of $\$ 400$ per month, plus a commission of $8 \%$ of gross sales;
Plan B: A salary of $\$ 610$ per month, plus a commission of $5 \%$ of gross sales.
For what amount of gross sales should Toni select plan A? Gross sales greater than $\$ 7000$
86. Wages. Eric can be paid for his masonry work in one of two ways:

Plan A: $\$ 300$ plus $\$ 9.00$ per hour;
Plan B: Straight $\$ 12.50$ per hour.
Suppose that the job takes $n$ hours. For what values of $n$ is plan B better for Eric? Values of $n$ greater than $85 \frac{5}{7} \mathrm{hr}$
87. Insurance Benefits. Under the "Green Badge" medical insurance plan, Carlee would pay the first $\$ 2000$ of her medical bills and $30 \%$ of all remaining bills. Under the "Blue Seal" plan, Carlee would pay the first $\$ 2500$ of bills, but only $20 \%$ of the rest. For what amount of medical bills will the "Blue Seal" plan save Carlee money? (Assume that her bills will exceed $\$ 2500$.) For more than $\$ 6000$
88. Checking Accounts. North Bank charges $\$ 10$ per month for a student checking account. The first 8 checks are free, and each additional check costs $\$ 0.75$. South Bank offers a student checking account with no monthly charge. The first 8 checks are free, and each additional check costs $\$ 3$. For what numbers of checks is the South Bank plan more expensive? (Assume that the student will always write more than 8 checks.) For more than 12 checks
89. Crude-Oil Production. The yearly U.S. production of crude oil $C(t)$, in millions of barrels, $t$ years after 2000 can be approximated by

$$
C(t)=-40.5 t+2159
$$

Determine (using an inequality) those years for which domestic production will drop below 1750 million barrels. Years after 2010
Source: U.S. Energy Information Administration
90. HDTVs. The percentage of U.S. households $p(t)$ with an HDTV $t$ years after 2005 can be approximated by

$$
p(t)=8 t+12.5
$$

Determine (using an inequality) those years for which more than half of all U.S. households will have an HDTV. Years after 2009
Source: Based on data from Consumer Electronics Association


- Answers to Exercises 91 and 92(a) are on p. IA-9.

91. Body Fat Percentage. The function given by

$$
F(d)=(4.95 / d-4.50) \times 100
$$

can be used to estimate the body fat percentage $F(d)$ of a person with an average body density $d$, in kilograms per liter.
a) A man is considered obese if his body fat percentage is at least $25 \%$. Find the body densities of an obese man.
b) A woman is considered obese if her body fat percentage is at least $32 \%$. Find the body densities of an obese woman.
92. Temperature Conversion. The function

$$
C(F)=\frac{5}{9}(F-32)
$$

can be used to find the Celsius temperature $C(F)$ that corresponds to $F^{\circ}$ Fahrenheit.
a) Gold is solid at Celsius temperatures less than $1063^{\circ} \mathrm{C}$. Find the Fahrenheit temperatures for which gold is solid.
b) Silver is solid at Celsius temperatures less than $960.8^{\circ} \mathrm{C}$. Find the Fahrenheit temperatures for which silver is solid. Fahrenheit temperatures less than $1761.44^{\circ}$
93. Manufacturing. Bright Ideas is planning to make a new kind of lamp. Fixed costs will be $\$ 90,000$, and variable costs will be $\$ 25$ for the production of each lamp. The total-cost function for $x$ lamps is

$$
C(x)=90,000+25 x
$$

The company makes $\$ 48$ in revenue for each lamp sold. The total-revenue function for $x$ lamps is

$$
R(x)=48 x
$$

(See Section 3.8.)
a) When $R(x)<C(x)$, the company loses money. Find the values of $x$ for which the company loses money. $\left\{x \left\lvert\, x<3913 \frac{1}{23}\right.\right\}$, or $\{x \mid x \leq 3913\}$
b) When $R(x)>C(x)$, the company makes a profit. Find the values of $x$ for which the company makes a profit. $\left\{x \left\lvert\, x>3913 \frac{1}{23}\right.\right\}$, or $\{x \mid x \geq 3914\}$
94. Publishing. The demand and supply functions for a locally produced poetry book are approximated by

$$
\begin{aligned}
& D(p)=2000-60 p \quad \text { and } \\
& S(p)=460+94 p
\end{aligned}
$$

where $p$ is the price, in dollars (see Section 3.8).
a) Find those values of $p$ for which demand exceeds supply. $\{p \mid p<10\}$
b) Find those values of $p$ for which demand is less than supply. $\{p \mid p>10\}$
95. How is the solution of $x+3=8$ related to the solution sets of

$$
x+3>8 \text { and } x+3<8 ?
$$

96. Why isn't roster notation used to write solutions of inequalities?

## SKILL REVIEW

To prepare for Section 4.2, review graphing equations and functions (Sections 2.2 and 2.3).

## Graph.

97. $y=2 x-3$
98. $y=-\frac{1}{3} x+4$ [2.2]
99. $y=2[2.3]$
100. $y=-4[2.3]$
101. $f(x)=-\frac{2}{3} x+1$
102. $g(x)=5 x-2[2.2]$

## SYNTHESIS

TN 103. The percentage of the U.S. population that owns an HDTV cannot exceed $100 \%$. How does this affect the answer to Exercise 90?
TW 104. Explain how the addition principle can be used to avoid ever needing to multiply or divide both sides of an inequality by a negative number.
Solve for $x$ and $y$. Assume that $a, b, c, d$, and $m$ are positive constants.
105. $3 a x+2 x \geq 5 a x-4$; assume $a>1\left\{x \left\lvert\, x \leq \frac{2}{a-1}\right.\right\}$
106. $6 b y-4 y \leq 7 b y+10$
107. $a(b y-2) \geq b(2 y+5)$; assume $a>2$
108. $c(6 x-4)<d(3+2 x)$; assume $3 c>d$
109. $c(2-5 x)+d x>m(4+2 x)$;
assume $5 c+2 m<d$
110. $a(3-4 x)+c x<d(5 x+2)$;
assume $c>4 a+5 d$
Determine whether each statement is true or false. If false, give an example that shows this.
111. For any real numbers $a, b, c$, and $d$, if $a<b$ and $\underset{\text { False; }}{c} d_{2}$ then $a-c \ll b-d$ and $4<5$, but $2-4=3-5$.
112. For all real numbers $x$ and $y$, if $x<y$, then $x^{2}<y^{2}$. False; $-3<-2$, but $9>4$.
97.

98.

99.

100.

101.

102.


$$
x<3 \text { and } x+\frac{1}{x}<3+\frac{1}{x}
$$

equivalent? Why or why not?
114. Are the inequalities

$$
x<3 \text { and } 0 \cdot x<0 \cdot 3
$$

equivalent? Why or why not?

## Solve. Then graph.

115. $x+5 \leq 5+x$ $\qquad$
116. $x+8<3+x$
117. $x^{2}>0 \quad\{x \mid x$ is a real number and $x \neq 0\} \longleftrightarrow \hat{0}_{x}^{x}$
118. Abriana rented a compact car for a business trip. At the time of rental, she was given the option of prepaying for an entire tank of gasoline at $\$ 3.099$ per gallon, or waiting until her return and paying $\$ 6.34$ per gallon for enough gasoline to fill the tank. If the tank holds 14 gal, how many gallons can she use and still save money by choosing the second option? About 6.8 gal or less
119. Refer to Exercise 118. If Abriana's rental car gets 30 mpg , how many miles must she drive in order to make the first option more economical?
More than 204 mi

- Try Exercise Answers: Section 4.1

11. (a) No; (b) no; (c) yes; (d) yes
12. $\xrightarrow[0]{\longrightarrow}\{y \mid y<6\},(-\infty, 6)$
13. $\{x \mid x>-1\}$, or $(-1, \infty) \underset{-10}{\leftrightarrows}$
14. $\{x \mid x<50\}$, or $(-\infty, 50) \xrightarrow[50]{\longrightarrow}$
15. $\{x \mid x \geq 2\}$, or $[2, \infty) \underset{-2}{\substack{1 \\-\\ ~}}$
16. $\left\{y \left\lvert\, y \leq-\frac{3}{2}\right.\right\}$, or $\left(-\infty,-\frac{3}{2}\right]$ 77. Lengths of time less than $7 \frac{1}{2} \mathrm{hr}$ 85. Gross sales greater than $\$ 7000$
17. $\left\{y \left\lvert\, y \geq-\frac{10}{b+4}\right.\right\} \quad$ 107. $\left\{y \left\lvert\, y \geq \frac{2 a+5 b}{b(a-2)}\right.\right\}$
18. $\left\{x \left\lvert\, x<\frac{4 c+3 d}{6 c-2 d}\right.\right\}$ 109. $\left\{x \left\lvert\, x>\frac{4 m-2 c}{d-(5 c+2 m)}\right.\right\}$
19. $\left\{x \left\lvert\, x<\frac{-3 a+2 d}{c-(4 a+5 d)}\right.\right\}$
